

AD-A227 433

DTIC FILE COPY

(1)

CCS Research Report No. 653

Information Theoretic Approach
To Geometric Programming

by

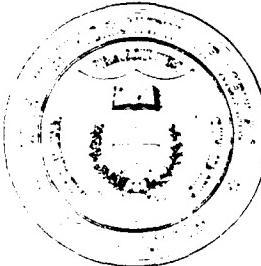
P. Brockett
A. Charnes

CENTER FOR
CYBERNETIC
STUDIES

The University of Texas
Austin, Texas 78712

DTIC
ELECTED
OCT. 04 1980
S B D

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited



20 70 01 005

(1)

CCS Research Report No. 653

Information Theoretic Approach
To Geometric Programming

by

P. Brockett
A. Charnes

May 1990

This research was partly supported by National Science Foundation Grant SES-8722504 with the Center for Cybernetic Studies, The University of Texas at Austin. Reproduction in whole or in part is permitted for any purpose of the United States Government.

CENTER FOR CYBERNETIC STUDIES

A. Charnes, Director

College of Business Administration, 5.202
The University of Texas at Austin
Austin, Texas 78712-1177
(512) 471-1821

DTIC
ELECTED
OCT. 4 1990
S B D

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

INFORMATION THEORETIC APPROACH TO GEOMETRIC PROGRAMMING

by

P. Brockett

and

A. Charnes

Abstract

This paper shows how the fundamental geometric inequality lemma of geometric programming can be obtained immediately from information theoretic methods. This results in a drastic simplification of the proof and points the way to other connections between information theory and geometric programming

Key Words

Geometric programming, information theory, geometric inequalities, duality.

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
<i>Per Form 50</i>	
Distribution	
Availability Codes	
Dist	Avail and/or Special
A-1	



Introduction

This note shows how information theoretic methods can be used to obtain simply the fundamental geometric inequality lemma. This lemma is the base used by Duffin, Peterson and Zener (1967) for proving their duality results for geometric programming, and its proof occupies three pages in their book. Their method of proof is a technically impressive procedure. However to illustrate the power of information theoretic methods we shall give a simple proof of this fundamental inequality. The approach we shall take is to show the connection between geometric programming and constrained Khinchine-Kullback-Leibler, or minimum discrimination information (MDI) estimation. Complete duality states for MDI estimation are known (c.f. Brockett, Charnes and Cooper (1980), Charnes, Cooper and Seiford (1978), Charnes and Cooper (1974a,b)). Additionally, in the usual duality state of interest, the statistical properties of the solution are known, facilitating sensitivity analysis. The computation of MDI estimation is easily performed using an unconstrained dual convex program involving only linear and exponential functions.

The information theoretic approach is based upon the mean information for discriminating between two densities f_1 and f_2 (relative to some fixed dominating measure λ). The mean information for discrimination in favor of f_1 against f_2 is defined by (c.f. Kullback (1959)).

$$I(f_1||f_2) = \int_0^{\infty} f_1(x) \ln \left[\frac{f_1(x)}{f_2(x)} \right] \lambda(dx).$$

Applying Jensen's inequality to the convex function $h(y) = y \ln y$ with the random variable $Y = \frac{f_1(X)}{f_2(X)}$, and assuming that X has probability measure $f_2(x) \lambda(dx)$ yields the result

$I(f_1|f_2) \geq 0$ with $I(f_1|f_2) = 0$ if and only if $f_1 = f_2$ a.s. [λ]. Amazingly, this is the only result needed to give a proof to the so-called geometric inequality (c.f. Duffin, Peterson and Zener (1967) pg 110). As usual, $0\ln 0$ is taken to be 0.

Lemma 1 (Geometric Inequality-Duffin, Peterson and Zener (1967))

Suppose $x \in \mathcal{R}^n$, and $\delta \in \mathcal{R}^n$ with $\delta_i \geq 0$, $i = 1, 2, \dots, n$.

Then

$$\sum x_i \delta_i + (\sum \delta_i) \ln(\sum \delta_i) \leq (\sum \delta_i) \ln(\sum \exp\{x_i\}) + \sum \delta_i \ln \delta_i$$

with equality if and only if there exists a non-negative number B such that

$$\delta_i = B \exp\{x_i\}, i = 1, 2, \dots, n.$$

Proof: Consider two probability distributions P and Q over $\{1, 2, \dots, n\}$ given by

$$p_i = \frac{\delta_i}{\sum \delta_i} \text{ and } q_i = \frac{\exp\{x_i\}}{\sum \exp\{x_i\}}.$$

Then since $I(P|Q) \geq 0$, and consequently

$$0 \leq (\sum \delta_i) I(P|Q) = \sum \delta_i \ln \left\{ \frac{\delta_i (\sum \exp\{x_i\})}{(\sum \delta_i) (\sum \exp\{x_i\})} \right\} = (\sum \delta_i) \ln (\sum \exp\{x_i\}) + \sum \delta_i \ln \delta_i -$$

$$(\sum \delta_i) \ln (\sum \delta_i) - \sum \delta_i x_i \text{ with equality if and only if } p_i = q_i, \text{ i.e., } \delta_i = B \exp\{x_i\}, i = 1, 2, \dots, n.$$

Q.E.D.

References:

Brockett, P.L., A. Charnes, and W.W. Cooper, "MDI Estimation via Unconstrained Convex Programming," Communications in Statistics B9 No. 3, pp. 223-234, (1980).

Charnes, A., and W.W. Cooper, "An Extremal Principle for Accounting Balance of a Resource-Value Transfer Economy; Existence, Uniqueness and Computation". Rendiconti di Accademia Nazionale dei Lincei, pp. 556-578, (1974a).

_____, _____, "Constrained Kullback-Leibler Estimation, Generalized Cobb-Douglas Balance, and Unconstrained Convex Programming". Rendiconti di Accademia Nazionale dei Lincei, Serir VIII, Vol. 56, April (1974b).

_____, _____, and Seiford, L., "Extremal Principles and Optimization Dualities for Kinchin-Kullback-Leibler Estimation," Mathematische Operationsforschung und Statistik, Vol. 9, No. 1. 1978, p. 21-29.